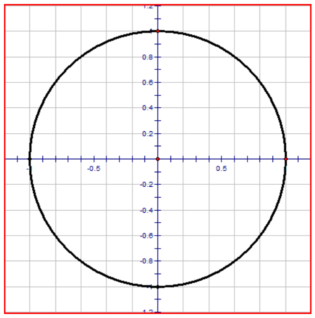
Right Triangles and Coordinates on the Unit Circle Learning Task:

1. The circle below is referred to as a "unit circle." Why is this the circle's name?



Part I

2. Using a protractor, measure a 30° angle with vertex at the origin, initial side on the positive x-axis and terminal side above the x-axis. Label the point where the terminal side intersects the circle as "A". Approximate the coordinates of point A using the grid.

3. Now, drop a perpendicular segment from the point you just put on the circle to the x-axis. You should notice that you have formed a right triangle. How long is the hypotenuse of your triangle? Using trigonometric ratios, specifically sine and cosine, determine the lengths of the two legs of the triangle. How do these lengths relate the coordinates of point A? How should these lengths relate to the coordinates of point A?

4. Using a Mira or paper folding, reflect this triangle across the y-axis. Label the resulting image point as point B. What are the coordinates of point B? How do these coordinates relate to the coordinates of point A? What obtuse angle was formed with the positive x-axis (the initial side) as a result of this reflection? What is the reference angle for this angle?

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5. Which of your two triangles can be reflected to create the angle in the third quadrant with a 30^o reference angle? What is this angle measure? Complete this reflection. Mark the lengths of the three legs of the third quadrant triangle on your graph. What are the coordinates of the new point on circle? Label the point C.

6. Reflect the triangle in the first quadrant over the x-axis. What is this angle measure? Complete this reflection. Mark the lengths of the three legs of the triangle formed in quadrant four on your graph. What are the coordinates of the new point on circle? Label this point D.

7. Let's look at what you know so far about coordinates on the unit circle. Complete the table.

θ	x-coordinate	y-coordinate

Notice that all of your angles so far have a reference angle of 30^o.

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Part II

8. Now, let's look at the angles on the unit circle that have 45^{o} reference angles. What are these angle measures?

9. Mark the first quadrant angle from #8 on the unit circle. Draw the corresponding right triangle as you did in Part I. What type of triangle is this? Use the Pythagorean Theorem to determine the lengths of the legs of the triangle. Confirm that these lengths match the coordinates of the point where the terminal side of the 45^{o} angle intersects the unit circle using the grid on your graph of the unit circle.

10. Using the process from Part I, draw the right triangle for each of the angles you listed in #8. Determine the lengths of each leg and match each length to the corresponding x- or y-coordinate on the unit circle. List the coordinates on the circle for each of these angles in the table.

θ	x-coordinate	y-coordinate

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Part III

11. At this point, you should notice a pattern between the length of the horizontal leg of each triangle and one of the coordinates on the unit circle. Which coordinate on the unit circle is given by the length of the horizontal leg of the right triangles?

12. Which coordinate on the unit circle is given by the length of the vertical leg of the right triangles?

13. Is it necessary to draw all four of the triangles with the same reference angle to determine the coordinates on the unit circle? What relationship(s) can you use to determine the coordinates instead?

14. Use your method from #13 to determine the (x, y) coordinates where each angle with a 60° reference angle intersects the unit circle. Sketch each angle on the unit circle and clearly label the coordinates. Record your answers in the table.

θ	x-coordinate	y-coordinate

Part IV

15. There are a few angles for which we do not draw right triangles even though they are very important to the study of the unit circle. These are the angles with terminal sides on the axes. What are these angles? What are their coordinates on the unit circle?

θ	x-coordinate	y-coordinate

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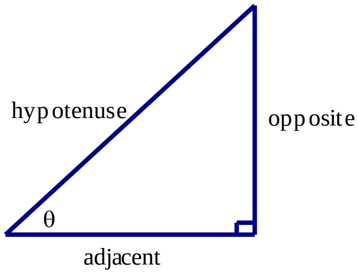
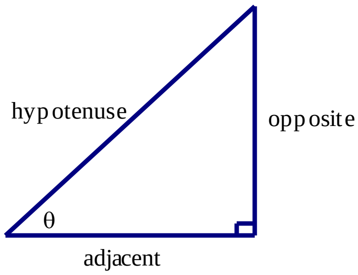
More Relationships in the Unit Circle Learning Task:

1. In Mathematics II, you learned three trigonometric ratios in relation to right triangles. What are these relationships?



2. There are three additional trigonometric ratios that you will use in this unit: secant, cosecant and cotangent.



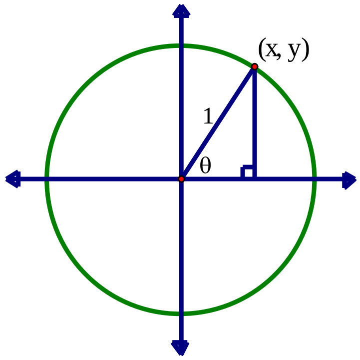
$$\sec\theta = \frac{hypotenuse}{adjacent}$$

$$\csc\theta = \frac{hypotenuse}{opposite}$$

$$\cot\theta = \frac{adjacent}{opposite}$$

How do these ratios relate to the trigonometric ratios from #1?

3. Moving the triangle onto the unit circle allows us to represent these six trigonometric relationships in terms of x and y. Express each of the six ratios in terms of x and y.



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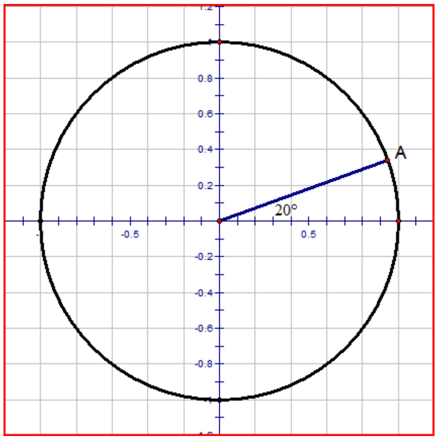
4. Based on these relationships x = _____________ and y = _____________. This is a special case of the general trigonometric coefficients (rcosθ, rsinθ) where r = 1.

5.



a. Use this relationship to determine the coordinates of A. Both coordinates a positive. Why is this true?

b. What angle would have coordinates (-0.9397, -0.3420) on the unit circle? Why?

c. What angle would have (0.9397, -0.3420) as its coordinates? Why?

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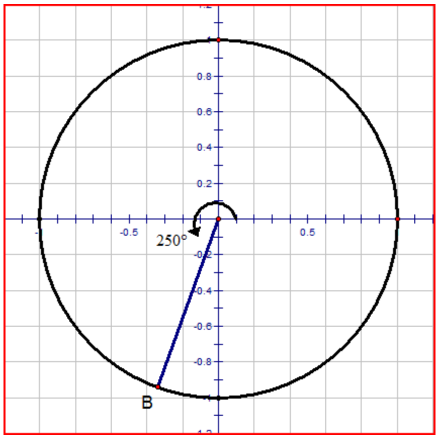
6.



a. What is the reference angle for 250^o?

b. What are the coordinates of this angle on the unit circle?

c. What 2^{nd} quadrant angle has the same reference angle? What are the coordinates of this angle on the unit circle?

7. Using a scientific or graphing calculator, you can quite easily find the sine, cosine and tangent of a given angle. This is not true for secant, cosecant, or cotangent. Remember from Mathematics II, that $\sin^{-1}$ is not the same as $\frac{1}{\sin\theta}$. Since the three new trigonometric ratios are not on a calculator, how can you use the definitions of the ratios from #2 to calculate the values?

8. A student entered sin30 in her calculator and got -0.98803. What went wrong?

9. Based on the graph of the unit circle on the grid, estimate each of the values. Do not use the trig keys on the calculator for this problem. You will need to use a protractor to mark each angle and then estimate the coordinates where the terminal side of the angle intersects the unit circle.

a. sec 60^o
b. csc 180^o
c. cot 235^o
d. sec -75^o
e. csc 490^o
f. cot 920^o

10. Use a calculator to find each of the following values.

a. sin 40^o
b. csc 40^o
c. cos 165^o
d. sec 165^o
e. tan 300^o
f. cot 300^o
g. csc 90^o
h. sec -140^o

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