UnWrapping the Unit Circle – Graphs from the Unit Circle Learning Task:

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Materials:
- Bulletin Board paper or butcher paper (approximately 8 feet long)
- Uncooked spaghetti
- Masking tape
- Protractor
- Meter stick
- Colored marker
- Yarn (about 7 feet long)

Part I: Unwrapping the Sine Curve

Tape the paper to the floor, and construct the diagram below. The circle's radius should be about the length of one piece of uncooked spaghetti. If your radius is smaller, break the spaghetti to the length of the radius. This is a unit circle with the spaghetti equal to one unit.

Using a protractor, make marks every 15° around the unit circle. Place a string on the unit circle at 0°, which is the point (1, 0), and wrap it counterclockwise around the circle. Transfer the marks from the circle to the string.

Transfer the marks on the string onto the *x*-axis of the function graph. The end of the string that was at 0° must be placed at the origin of the function graph. Label these marks on the *x*-axis with the related angle measures from the unit circle (e.g., 0°, 15°, 30°, etc.).

1. What component from the unit circle do the *x*-values on the function graph represent?

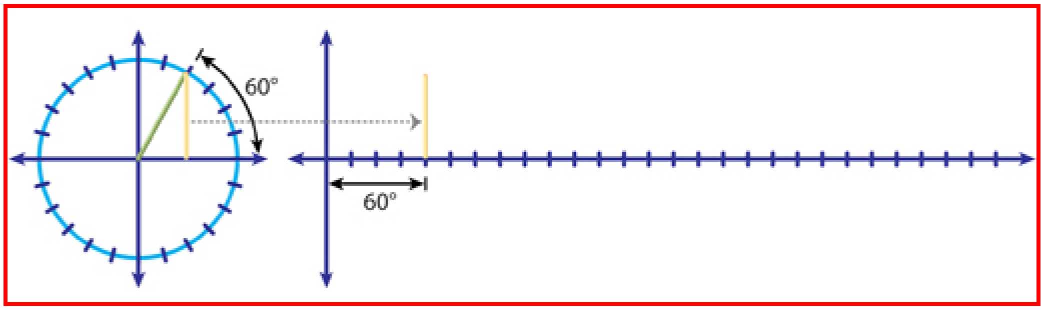
x-values = ________________________

Use the length of your spaghetti to mark one unit above and below the origin on the *y*-axis of the function graph. Label these marks 1 and –1, respectively.

Draw a right triangle in the unit circle where the hypotenuse is the radius of the circle to the 15° mark and the legs lie along and perpendicular to the *x*-axis.

Break a piece of spaghetti to the length of the vertical leg of this triangle, from the 15° mark on the circle to the *x*-axis. Let this piece of spaghetti represent the *y*-value for the point on the function graph where *x* = 15°. Place the spaghetti piece appropriately on the function graph and make a dot at the top of it. **Note:** Since this point is above the *x*-axis in the unit circle, the corresponding point on the function graph should also be above the *x*-axis.

Transferring the Spaghetti for the Triangle Drawn to the 60° Mark



Continue constructing triangles and transferring lengths for all marks on the unit circle. After you have constructed all the triangles, transferred the lengths of the vertical legs to the function graph, and added the dots, draw a smooth curve to connect the dots.

2. The vertical leg of a triangle in the unit circle, which is the *y*-value on the function graph, represents what function of the related angle measure?

   y-values = ________________________

Label the function graph you just created on your butcher paper $y = \sin x$.

3. What is the period of the sine curve? That is, what is the wavelength? After how many radians does the graph start to repeat? How do you know it repeats after this point?

4. What are the zeroes of this function? (Remember: The *x*-values are measuring angles and zeroes are the *x*-intercepts.)

5. What are the *x*-values at the maxima and minima of this function?

6. What are the *y*-values at the maxima and minima?

7. Imagine this function as it continues in both directions. Explain how you can predict the value of the sine of 390°.

8. Explain why sin 30° = sin 150°. Refer to both the unit circle and the graph of the sine curve.

Part II: Unwrapping the Cosine Curve

You used the length of the vertical leg of a triangle in the unit circle to find the related *y*-value in the sine curve. Determine what length from the unit circle will give you the *y*-value for a cosine curve. Using a different color, create the graph on your butcher paper and label it $y = \cos x$.

9. In what ways are the sine and cosine graphs similar? Be sure to include a discussion of intercepts, maxima, minima, and period.

10. In what ways are the sine and cosine graphs different? Again, be sure to include a discussion of intercepts, maxima, minima, and period.

11. Will sine graphs continue infinitely in either direction? How do you know? Identify the domain and range of $y = \sin x$.

12. Will cosine graphs continue infinitely in either direction? How do you know? Identify the domain and range of $y = \cos x$.