

Domain and Range of Continuous Functions

Shake, Rattle and Roll

ACTIVITY

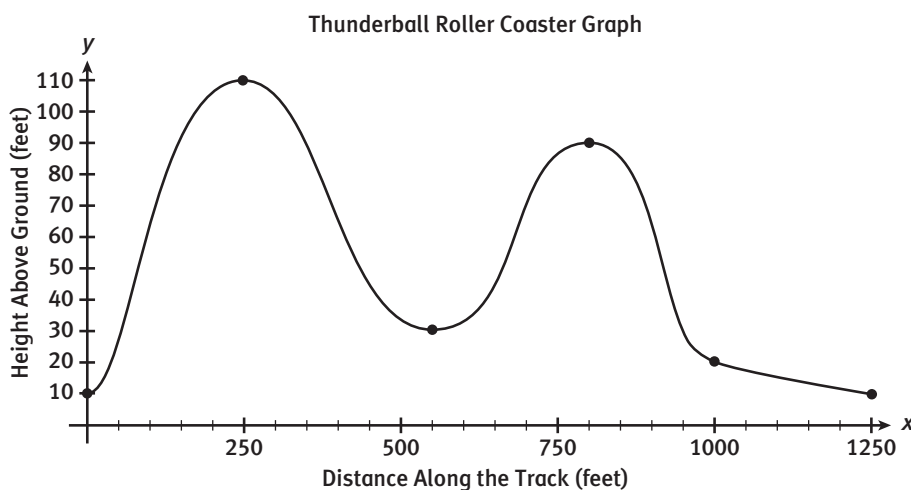
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SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Visualization, Interactive Word Wall

My Notes

Roller coasters are scary and fun to ride. Wooden roller coasters shake and rattle as part of the thrill of the ride. Below is the graph of the heights reached by the cars of the wooden roller coaster, Thunderball, over its first 1250 feet of track. The graph displays a function because each input value has one and only one output value. You can see this visually using the **vertical line test**.

Study this graph to determine the domain and range.



The domain gives all values of the **independent variable**: distance along the track in feet. These values are graphed along the horizontal or x -axis.

The domain can be written in set notation as:

$$\{\text{all real values of } x: 0 \leq x \leq 1250\}$$

Read this notation as: *the set of all real values of x , between 0 and 1250, inclusive.*

The range gives the values of the **dependent variable**: height above the ground in feet. The values are graphed on the vertical or y -axis.

The range can be written in set notation as:

$$\{\text{all real values of } y: 10 \leq y \leq 110\}$$

Read this notation as: *the set of all real values of y , between 10 and 110, inclusive.*

The graph above shows data that are **continuous**. The points in the graph are connected, indicating that domain and range are sets of real numbers with no breaks in between. A graph of **discrete** data consists of individual points that are not connected by a line or curve.

MATH TERMS

The **vertical line test** is a visual check to see if a graph represents a function. For a function, every vertical line drawn in the coordinate plane will intersect the graph in at most one point. This is equivalent to having each domain element associated with one and only one element of the range.

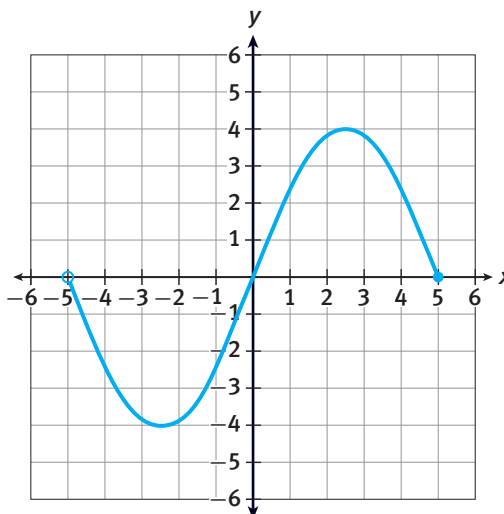
ACADEMIC VOCABULARY

An **independent variable** is the variable for which input values are substituted in a function. A **dependent variable** is the variable whose value is determined by the input or value of the independent variable.

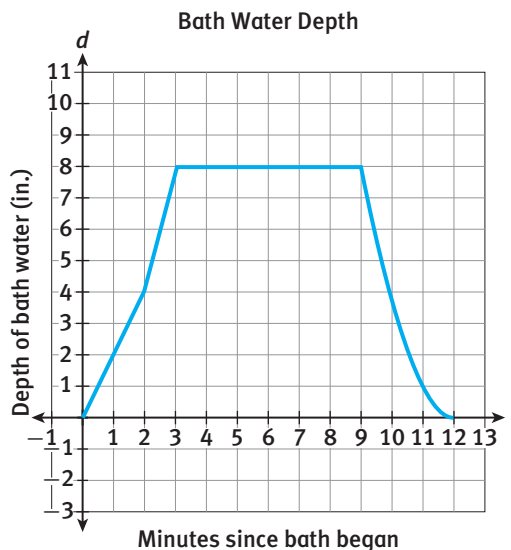
SUGGESTED LEARNING STRATEGIES: Group Discussion

My Notes

- 1a.** Use set notation to write the domain and range for the graph below. Does this graph appear to represent a function? Justify your answer. Are the data discrete or continuous? Why?



- 1b.** The graph below shows the relationship between t , the length of time of the bath (from the time water starts running through the time the tub is drained) and d , the depth of the water in the bath tub. The graph represents function d (bath water depth). What are the dependent and independent variables? Explain. Use set notation to write the domain and range of function d . Are the data discrete or continuous and why?

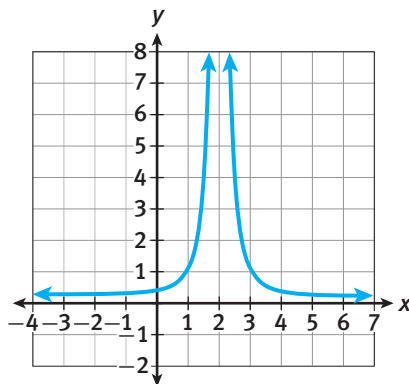


SUGGESTED LEARNING STRATEGIES: Marking the Text, Questioning the Text, Think Aloud

My Notes

EXAMPLE

Give the domain and range of the function $f(x) = \frac{1}{(x-2)^2}$ graphed below.



Step 1: Study the graph.

The sketch of this graph is a portion of the function represented by the equation $f(x) = \frac{1}{(x-2)^2}$.

Step 2: Look for values for which the domain causes the function to be undefined. Look how the graph behaves near $x = 2$.

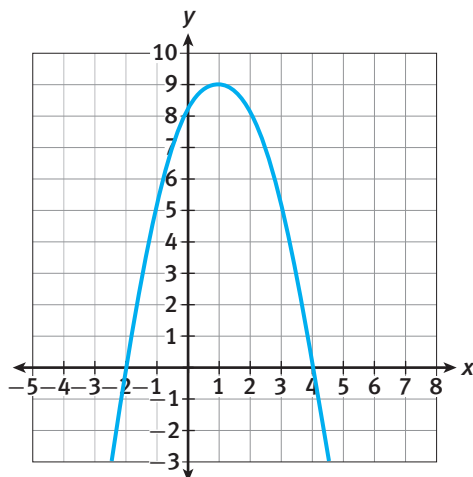
Solution: The domain and range for $f(x) = \frac{1}{(x-2)^2}$ can be written:

Domain: {all real values of x : $x \neq 2$ }

Range: {all real values of y : $y > 0$ }

TRY THESE

a. Give the domain and range of the function $f(x) = 8 + 2x - x^2$ graphed below.



MATH TIP

Notice the result when $x = 2$ is substituted into $f(x)$.

$$f(2) = \frac{1}{(2-2)^2} = \frac{1}{0}$$

Division by zero is undefined in mathematics.

My Notes

SUGGESTED LEARNING STRATEGIES: Create Representations, Group Discussion

TRY THESE (continued)

- b.** Give the domain and range for the equation $y = 2x - 1$. Explain whether this equation represents a function and how you determined this.

MATH TIP

The domain is restricted to avoid situations where division by zero or taking the square root of a negative number would occur.

Technology Time

- Work with a partner to investigate the equations listed in the chart using graphing technology. Every equation given here is a function.
- Determine the domain and range for each function from the possibilities listed below the chart.
- Select the appropriate domain from choices 1–6 and record your answer in the Domain column. Then select the appropriate range from choices a–f and record the appropriate range in the Range column.
- When the chart is complete, compare your answers with those from another group.

	Function	Domain	Range
1.	$y = -3x + 4$		
2.	$y = x^2 - 6x + 5$		
3.	$y = 9x - x^3$		
4.	$y = x + 1 $		
5.	$y = 3 + \sqrt{x}$		
6.	$y = \frac{4}{x}$		

Possible Domains:

- 1) all real numbers
- 2) all real x , such that $x \neq -2$
- 3) all real x , such that $x \neq 0$
- 4) all real x , such that $x \neq 2$
- 5) all real x , such that $x \geq 0$
- 6) all real x , such that $x \leq 0$

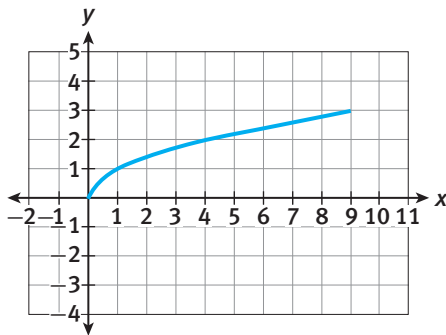
Possible Ranges:

- a) all real numbers
- b) all real y , such that $y \neq 0$
- c) all real y , such that $y \geq -4$
- d) all real y , such that $y \geq 0$
- e) all real y , such that $y \geq 1$
- f) all real y , such that $y \geq 3$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

- Give the domain and range for the function graphed below. Explain why this graph represents a function.

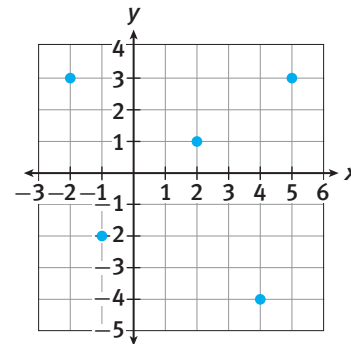


- A student calculates how far away a lightning strike is, based on when the thunder is heard. The student makes the table below using $\frac{1}{3}$ km/sec as the average speed of sound under rainy conditions. If the thunder is only heard when the lightning strike is within 15 km of the listener, what are the domain and range for this model? Is this relation a function? How do you know?

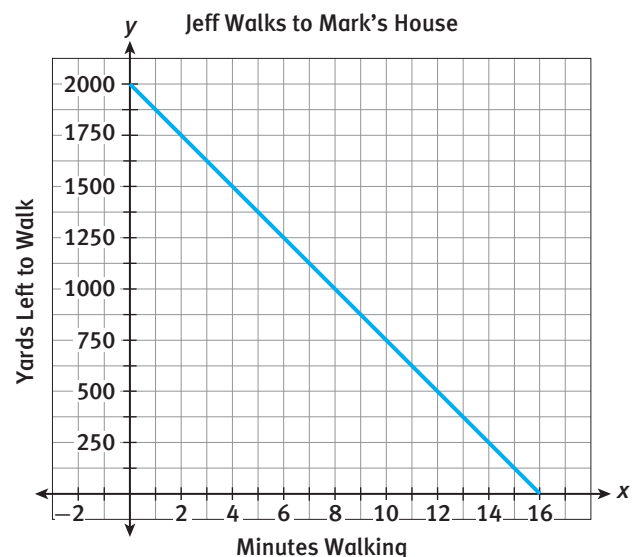
Time until thunder is heard (sec)	1	2	3	4	5	6
Distance from lightning strike (km)	$\frac{1}{3}$	$\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2

- Give the domain and range of the function $f(x) = -4x - 5$.

- The graph below shows five points that make up the function h . Give the domain and the range for the function h .



- Jeff walks at an average rate of 125 yards per minute. Mark's house is located 2000 yards from Jeff's house. The graph below shows how far Jeff still needs to walk to reach Mark's house. Give the domain and range for this model. Is this model a function? Explain.



CHECK YOUR UNDERSTANDING (continued)

- Capital letters sketched in the coordinate plane may or may not be functions. Pick one letter that represents a function and two that do not. Use the vertical line test as part of the explanation for your selections.
- MATHEMATICAL REFLECTION** Describe at least three different methods for determining if a relation is a function. Which method do you prefer and why?